Failure Analysis

Failure occurs in a variety of fashions

- Shear failure
- Tensile (normal) failure
- Bending
- Compressive (crushing)
- Buckling of thin columns
- Fatigue (all of the above)

These are macroscale failures – things look different at a point.

Shear Failure

- Punches rely on shear failure to knock out a slug and create a hole
- Pins that hold shafts together can shear
- Keys can shear
- Failure in torsion is usually a shear failure.

Normal Failure

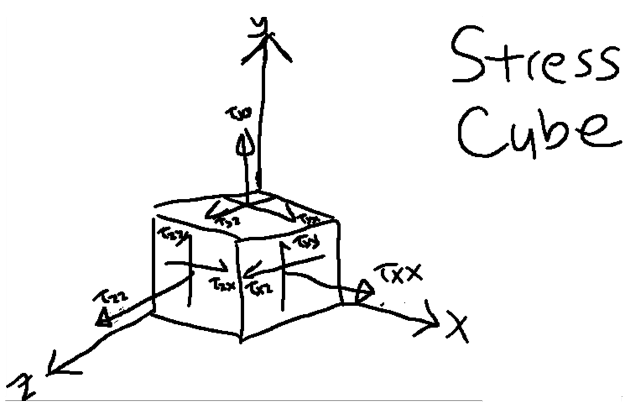
- Dog-bone tension test is a normal failure
- Fastener heads can be popped off in a normal failure
- Compressive failure occurs when the opposite load exceeds the materials yield strength and rupture usually results

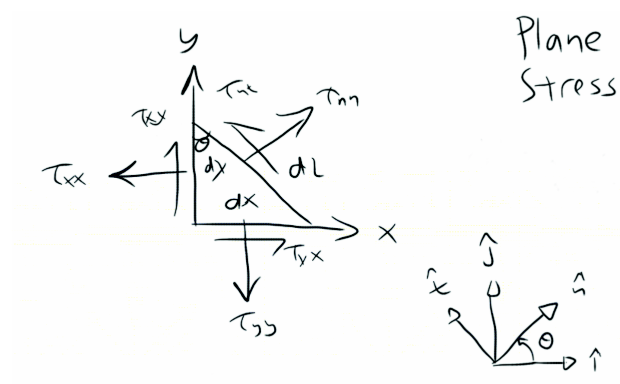
Bending Failure

- When an element is placed into bending, failure can occur at the top or bottom surface through (usually) a tensile failure or (rarely) a compressive failure.

Static Equilibrium Inside Material

- If a chunk of material is cut away, then forces along the faces of the material must exist to maintain the material's equilibrium state.
- The positive traction vectors are shown on the faces of the stress cube.
- Quasi-static assumption would say that sum of forces is zero on the stress cube





Normal and Tangential Stress

$$\hat{n} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$
$$\hat{t} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\cos\theta\,\hat{n} - \sin\theta\,\hat{t} = \hat{i}$$
$$\sin\theta\,\hat{n} + \cos\theta\,\hat{t} = \hat{j}$$

$$\hat{u}_1^T = [\hat{i}\ \hat{j}\ \hat{k}]$$
$$\vec{dA}_x = dy\,dz\,(-\hat{i})$$
$$\vec{dA}_y = dx\,dz\,(-\hat{j})$$
$$\hat{u}_2^T = [\hat{n}\ \hat{t}\ \hat{k}]$$
$$\vec{dA} = dL\,dz\,\hat{n}$$

traction vectors

$$\vec{t}(-\hat{i}) = \vec{dA}_x \cdot \vec{\tau}$$

$$\vec{dA}_x \cdot \hat{u}_1^T \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \hat{u}_1$$

$$\vec{dA}_x \cdot \hat{u}_1^T = -dy\,dz\,\hat{i} \cdot [\hat{i}\ \hat{j}\ \hat{k}] = [-dy\,dz\ \ 0\ \ 0]$$

$$\vec{t}(-\hat{i}) = [-dy\,dz\ \ 0\ \ 0] \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

$$= [-\tau_{xx}\,dy\,dz\ \ -\tau_{xy}\,dy\,dz\ \ -\tau_{xz}\,dy\,dz] \begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix}$$

$$\vec{t}(-\hat{i}) = -dy\,dz\,(\tau_{xx}\hat{i} + \tau_{xy}\hat{j} + \tau_{xz}\hat{k})$$
$$\vec{t}(-\hat{j}) = -dx\,dz\,(\tau_{yx}\hat{i} + \tau_{yy}\hat{j} + \tau_{yz}\hat{k})$$

Static Equilibrium

$$\underline{\tau} = \hat{u}_1^T \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \hat{u}_1 = \hat{u}_2^T \begin{bmatrix} \tau_{nn} & \tau_{nt} & \tau_{nz} \\ \tau_{tn} & \tau_{tt} & \tau_{tz} \\ \tau_{zn} & \tau_{zt} & \tau_{zz} \end{bmatrix} \hat{u}_2$$

$$t(\hat{n}) = dl\,dz(\tau_{nn}\hat{n} + \tau_{nt}\hat{t} + \tau_{nz}\hat{k})$$

Static Equilibrium – Plane Stress (only $\hat{i},\hat{j}/\hat{n},\hat{t}$ plane)

$$\vec{t}(-\hat{i}) + \vec{t}(-\hat{j}) + \vec{t}(\hat{n}) = \vec{0}$$

- factor out dz

$$(\tau_{xx}\hat{i} + \tau_{xy}\hat{j})\,dy + (\tau_{yx}\hat{i} + \tau_{yy}\hat{j})\,dx = (\tau_{nn}\hat{n} + \tau_{nt}\hat{t})\,dl$$

- $\hat{i},\hat{j} \Rightarrow \hat{n},\hat{t}$; $\sin\theta = \frac{dx}{dl}$, $\cos\theta = \frac{dy}{dl}$

$$\tau_{nn} = \tau_{xx}\cos^2\theta + \tau_{xy}\sin\theta\cos\theta + \tau_{yx}\cos\theta\sin\theta + \tau_{yy}\sin^2\theta$$

$$\tau_{nt} = -\tau_{xx}\sin\theta\cos\theta + \tau_{xy}\cos^2\theta - \tau_{yx}\sin^2\theta + \tau_{yy}\cos\theta\sin\theta$$

- double angle formulae:

$$\tau_{nn} = \tfrac{1}{2}(\tau_{xx}+\tau_{yy}) + \tfrac{1}{2}(\tau_{xx}-\tau_{yy})\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\tfrac{1}{2}(\tau_{xx}-\tau_{yy})\sin 2\theta + \tau_{xy}\cos 2\theta$$

Max Normal Stress

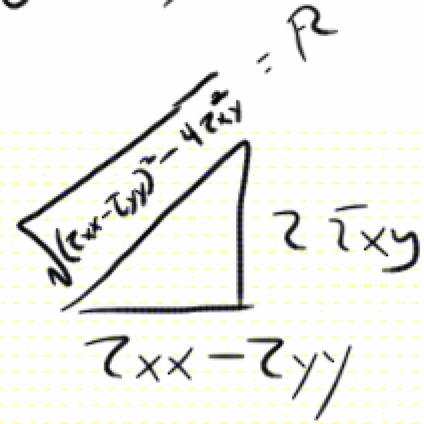
maximize τ_{nn},

$$\frac{d\tau_{nn}}{d\theta} = \phi = 2\tau_{xy}\cos 2\theta - (\tau_{xx} - \tau_{yy})\sin 2\theta$$

$$\cos 2\theta = \frac{\tau_{xx} - \tau_{xy}}{2\tau_{xy}}\sin 2\theta \Rightarrow$$

$$\tau_{nt} = \phi$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\tau_{xx} - \tau_{yy}}$$



$$\tau_{nn} = \frac{1}{2}(\tau_{xx} + \tau_{yy}) + \frac{1}{2}\frac{(\tau_{xx} - \tau_{yy})^2}{R} + \frac{2\tau_{xy}^2}{R}$$

Maximum Shear

maximize shear

$$\frac{d\tau_{nt}}{d\theta} = \phi = -(\tau_{xx} - \tau_{yy})\cos 2\theta - 2\tau_{xy}\sin 2\theta$$

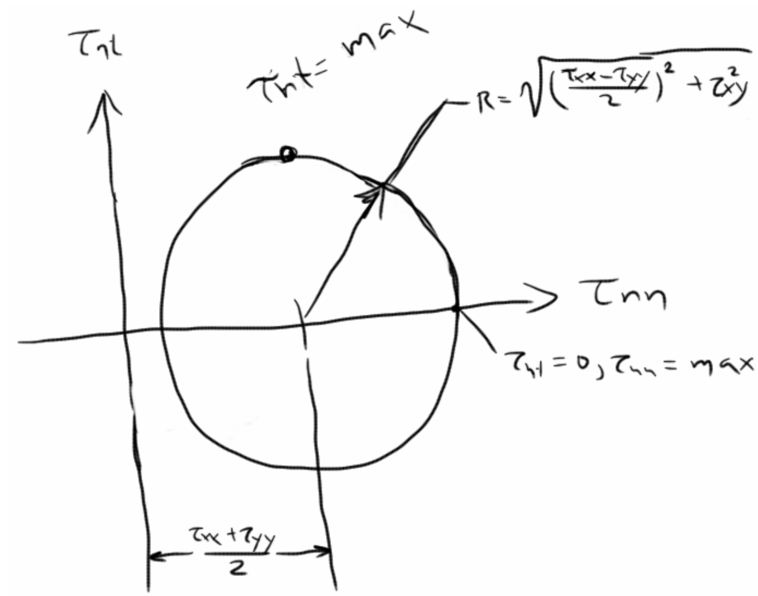
$$\tan 2\theta = -\frac{\tau_{xx} - \tau_{yy}}{2\tau_{xy}} \Rightarrow \sin 2\theta = -\frac{\tau_{xx} - \tau_{yy}}{R}$$

$$\cos 2\theta = \frac{2\tau_{xy}}{R}$$

$$\tau_{nt} = \frac{1}{2}\frac{(\tau_{xx} - \tau_{yy})^2}{R} + 2\frac{\tau_{xy}^2}{R}$$

Mohr's Circle

- Mohr's circle tells us that every element in a material experiences some stress level
- By rotating the cutting plane for the same stress element, different equilibrium stresses can be determined.
- Because the stress tensor is symmetric for most loading conditions, there will be a maximum normal stress.
- The stress tensor can be realigned so that a maximum shear stress can be determined.



Failure Along Maximum Stress

- Several failure theories in the literature (Tresca and von Mises being the most prevalent in undergraduate texts)
- All failure theories assume that hydrostatic stress (think uniform pressure) does not affect failure.
- All failure theories assume that failure occurs because grain boundaries are shifted along lines of maximum shear stress or distortion.
- Tresca assumes the maximum shear stress. Von Mises assumes maximum distortion energy.
- Failure is most likely when the material is weakened, through prior plastic deformation, fatigue, material defect, or stress concentration.

Mohr's Circle for 3D Stress

- Mohr's circle can be extended to 3D stress state.
- Just do the zx plane and the yz plane, in addition to the xy plane
- Principal stresses can also be gained by diagonalizing the stress tensor, which can be done for any symmetric tensor
- Matlab will do it in an instant. But, if you want to find eigenvalues and eigenvectors, knock yourself out.

Shear Stress Test

- A torsion test on a thin walled cylinder puts an element into a state of pure shear
- Maximum shear is $\frac{1}{2}(\tau_{max}-\tau_{min})=k_T$
- If the element yields in shear at a stress, S, then $S=k_T$
- NOTE: Mohr's circles and stress elements will be provided as soon as UALR's PoS network lets me email off my iPad.
- For now, copy off white-board.

Tension Test Result

- Don't usually have shear strength for materials
- What is the maximum shear stress when yield occurs? $\frac{1}{2}(\sigma_y - 0) = k_T$
- Therefore, $k_T = \frac{1}{2}\sigma_y$
- Tresca criterion in terms of normal yield stress is $\frac{1}{2}(\tau_{max} - \tau_{min}) = \sigma_y$
- This is a useful estimate of loading that will yield plastic deformation for a general 3D stress on elements.

Fatigue

- ASTM - “progressive localized permanent structural change due to fluctuating stress”
- $\tau(t) = \tau_m + \tau_a \sin(\omega t + \phi)$
- Even if stress remains below elastic limit, failure will occur for some cyclic stress amplitude
- If there is an amplitude below which the material can withstand 10^8 cycles

Mean Stress and Fatigue

- Mean stress affects endurance limit as well as amplitude
- As mean stress increases, fatigue limit decreases
- NOTE: the scatter in fatigue data is enormous, so some kind of statistical reliability can be quoted, but not an absolute reliability.
- NOTE: stress concentrations amplify the effects of fatigue.

Fatigue Mechanisms

- Ductile materials … at the very small level in a material, there are little bitty stress concentrations
- When a material is loaded, slip (plastic deformation) occurs at these local high stress points.
- Plastic deformation on unloading leads to work-hardening, which represents the accumulation of plastic strain.
- Cracks form, increasing the stress concentration
- Rinse and repeat about a million times and these cracks will tend to grow.