Elements of Mechanical Design Homework 1
Assigned Jan. 24, 2014, Due: Jan. 29, 2014

1. For a 20 tooth, 20 pitch, 20 degree pressure angle brass gear in mesh with a 40 tooth, 20 pitch, 20 degree pressure angle steel gear, what is the optimal axle spacing? What is the gear ratio between these two gears?

2. You have an input gear with 15 teeth and an output gear with 30 teeth. If the motor torque is $150\, N \cdot m$, what is the output torque (assuming no losses) of the gear-box. If the input speed is $3000\, rpm$, what is the output speed?

3. You have a two-stage gearbox with a 10 tooth gear meshing with a 25 tooth gear and a 12 tooth gear on the same shaft as the 25 tooth gear meshing with a 30 tooth gear. If the motor torque is $100\, N \cdot m$, what is the output torque (assuming no losses) of the gear-box. If the input speed is $15000\, rpm$, what is the output speed?

Formulae

$$\sigma = \frac{54 W_t P_d}{\pi^2 F}$$

$$\omega_o = \frac{Z_i}{Z_o} \omega_i \qquad \tau_o = \frac{Z_o}{Z_i} \tau_i$$

$$W_t = \frac{2 \tau_i P_d}{Z_i} \qquad L = \frac{Z_i + Z_o}{2 P_d}$$

Nomenclature

P_d diametral pitch in teeth per inch

Z_i number of teeth on the i^{th} gear

φ pressure angle in degrees or radians

τ_i input torque, in $N \cdot m$

τ_o output torque, in $N \cdot m$

W_t tooth contact force, tangential to pitch circle in N

W_n tooth contact force, normal to pitch circle in N = $W_t \tan(\varphi)$

F gear face width, in mm

σ normal bending stress at tooth root, in Pa

σ_y normal yield stress, in Pa

ω_o output speed in $\frac{rad}{sec}$

ω_i input speed in $\frac{rad}{sec}$

L optimal gear spacing, in mm

Conversions

$1\, inch = 25.4\, mm$

The forces acting on the axles (assuming 4 stages) are:

stage 1 axle: $\vec{B}_1 + W_{t1} \hat{i} + W_{n1} \hat{j} = \hat{0}$

stage 2 axle: $\vec{B}_2 - W_{t1} \hat{i} - W_{n1} \hat{j} + W_{t2} \hat{i} + W_{n2} \hat{j} = \hat{0}$

stage 3 axle: $\vec{B}_3 - W_{t2} \hat{i} - W_{n2} \hat{j} + W_{t3} \hat{i} + W_{n3} \hat{j} = \hat{0}$

stage 4 axle: $\vec{B}_4 - W_{t3} \hat{i} - W_{n3} \hat{j} = \hat{0}$

The magnitudes can be calculated (remembering the relation between W_t and W_n) as:

$\|\vec{B}_1\| = W_{t1} sec(\varphi)$, $\|\vec{B}_2\| = (W_{t1} - W_{t2}) sec(\varphi)$, $\|\vec{B}_3\| = (W_{t2} - W_{t3}) sec(\varphi)$, $\|\vec{B}_1\| = W_{t3} sec(\varphi)$