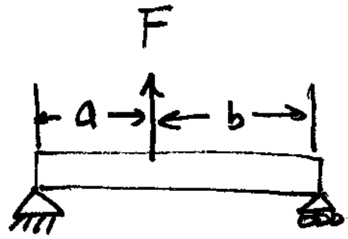
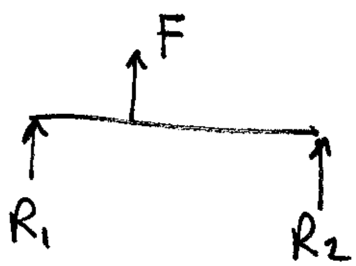
Beam deflection — simple supports, intermediate load
A-9, entry 6



$L = a + b$

$L - a = b \qquad b - L = -a$

FBD



$\Sigma F_y = \emptyset = R_1 + R_2 + F \Rightarrow R_2 = -F - R_1$

$\Sigma M_L = \phi = -R_1 L - F b \Rightarrow R_1 = -\frac{b}{L} F$

$R_2 = \left(-1 + \frac{b}{L}\right) F$

$R_2 = -\frac{a}{L} F$

$R_1 = V(0^-)$

$q(x) = F\,\delta(x-a)$

$V(x) = V(0^-) + F\,u(x-a) = \left(-\frac{b}{L} + u(x-a)\right) F$

$M(x) = \underbrace{M(\cancel{0^-})}_{0} + \left(-\frac{b}{L}x + r(x-a)\right) F = EI \frac{d^2y}{dx^2}$

$EI \frac{dy}{dx} = \theta(0^-) + \left(-\frac{1}{2}\frac{b}{L}x^2 + p(x-a)\right) F = \frac{d\theta}{dx}$

$EI\, y(x) = \left(-\frac{1}{6}\frac{b}{L}x^3 + c(x-a)\right) F + \theta(0^-)x + \overset{\emptyset}{\cancel{y(0^-)}}$

$EI\, y(0) = \phi = y(0^-)$

$EI\, y(L) = \phi = \theta(0^-) L + \left(-\frac{1}{6}\frac{b}{L}L^3 + \overset{\frac{1}{6}b^3}{c(\underbrace{L-a}_{b})}\right) F$

$\delta(x) = \begin{cases} 1 & x = \phi \\ 0 & \text{else} \end{cases}$, $u(x) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{else} \end{cases}$, $r(x) = \begin{cases} x & x \geq 0 \\ 0 & \text{else} \end{cases}$, $p(x) = \begin{cases} \frac{1}{2}x^2 & x \geq 0 \\ 0 & \text{else} \end{cases}$, $c(x) = \begin{cases} \frac{1}{6}x^3 & x \geq 0 \\ 0 & \text{else} \end{cases}$

$$\theta(0^-)L + \frac{-FbL^2}{6} + \frac{1}{6}Fb^3 = \phi$$

$$\theta(0^-) = \frac{1}{6}Fb\left(L - \frac{b^2}{L}\right) = \frac{1}{6}\frac{Fb}{L}(L^2 - b^2)$$

$$EI\,y(x) = F\,c(x-a) + \frac{1}{6}\frac{Fb}{L}(L^2-b^2)x - \frac{1}{6}\frac{Fb}{L}x^3$$

$$EI\,y(x) = F\,c(x-a) + \frac{1}{6}\frac{Fb}{L}x\,(L^2-b^2-x^2)$$

$$x < a \Rightarrow c(0) = \phi \Rightarrow y(x) = \frac{1}{6}\frac{Fb}{EIL}x\,(L^2-b^2-x^2)$$

that's y_{AB} from table A-9-6, with $F = -F$, since F is "down positive" in the table

$$x > a \Rightarrow c(x-a) = \frac{1}{6}(x-a)^3$$

$$EI\,y(x) = \frac{1}{6}F(x-a)^3 + \frac{1}{6}\frac{Fb}{L}x\,(L^2-b^2-x^2)$$

$$b = L-a \Rightarrow b^2 = L^2 - 2aL + a^2 \Rightarrow L^2 - b^2 = 2aL - a^2$$

$$EI\,y(x) = \frac{1}{6}\frac{F}{L}\left[L(x-a)^3 + (L-a)x\,(2aL-a^2-x^2)\right]$$

$$= \frac{F}{6L}\left[L(x^3 - 3ax^2 + 3a^2x - a^3) + 2aL^2x - 3a^2Lx + a^3x - (L-a)x^3\right]$$

$$= \frac{F}{6L}\left(ax^3 - 3aLx^2 + (3a^2L + 2aL^2 + a^3 - 3a^2L)x - a^3L\right]$$

$$= \frac{Fa}{6L}\left(x^3 - 3Lx^2 + (2L^2+a^2)x - a^2L\right)$$

$$(x-L)(x^2+a^2-2Lx) = x^3 - 3Lx^2 + (2L^2+a^2)x - a^2L$$

$$y(x) = \frac{Fa(x-L)}{6EIL}(x^2+a^2-2Lx)$$